MATH 207(c)–Analysis IV

1. Compute the Laplace transform (and find an abscissa of convergence) of the shifted Heaviside function $H_a : [0, +\infty) \to \mathbb{R}$,

$$H_a(t) = \begin{cases} 0, & t \leqslant a, \\ 1, & t \geqslant a. \end{cases}$$

2. Compute the Laplace transforms of the modulated hyperbolic trigonometric functions:

$$f(t) = e^{-at} \cosh(\omega t), \quad g(t) = e^{-at} \sinh(\omega t).$$

3. In each of the following cases, find a function $f : [0, +\infty) \to \mathbb{C}$ such that $\mathcal{L}[f](z) = F(z)$. You may use the Laplace transforms of the explicit examples we have computed in class or in previous exercises.

(a) $F(z) = \dfrac{4z}{z^2 + 64}$,

(b) $F(z) = \dfrac{z}{(z+1)(z+2)}$,

(c) $F(z) = \dfrac{1}{z^3 + z}$.

(Hint: You might want to use some algebra to split the above expressions into sums of functions for which you can recognise their Laplace transforms.)

4. Using the residue theorem, compute the inverse Laplace transform of the following functions:

(a) $F(z) = \frac{1}{(z+1)^2(z+2)}$,

(b) $F(z) = \frac{z^2}{(z^2+1)^2}$.

5. Solve the 2^{nd} order initial value problem

$$\begin{cases} y''(x) + 2y'(x) + y(x) = f(x) & \text{for } x > 0, \\ y(0) = 1, \quad y'(0) = 1 \end{cases}$$

in the following two cases:

(a) $f(t) = 0$,

(b) $f(t) = t$.

6. Solve the 3^{rd} order initial value problem

$$\begin{cases} y'''(t) + y'(t) = te^{-t} & \text{for } t > 0, \\ y(0) = 0, \quad y'(0) = 0, \quad y''(0) = 1. \end{cases}$$